Sisyphus effect in neural networks with plasticity

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Summary

- The Model
  - Neural network composed of leaky integrate-and-fire (LIF) neurons
  - Neurons are pulse-coupled with excitatory coupling (synapses)
  - Plastic Synapses: Spike-Timing Dependent Plasticity (STDP)
- Main Results
  - The collective dynamics exhibits irregular oscillations between strongly and weakly synchronized states
  - The oscillations are due to a completely deterministic mechanism: the Sisyphus Effect (SE)
  - This effect is induced by the feedback between the modifications of the synaptic weights and the level of synchronization of the neurons
  - The SE generates endless oscillations in the equilibrium values of the synaptic weights, and this prevents the system from relaxing into a stationary macroscopic state.

Sisyphus Myth

Tiziano , Museo Nacional del Prado

Sisyphus was the mythological king of Corinth compelled to roll a heavy boulder up a hill, only to watch it roll back down as it approaches the top.

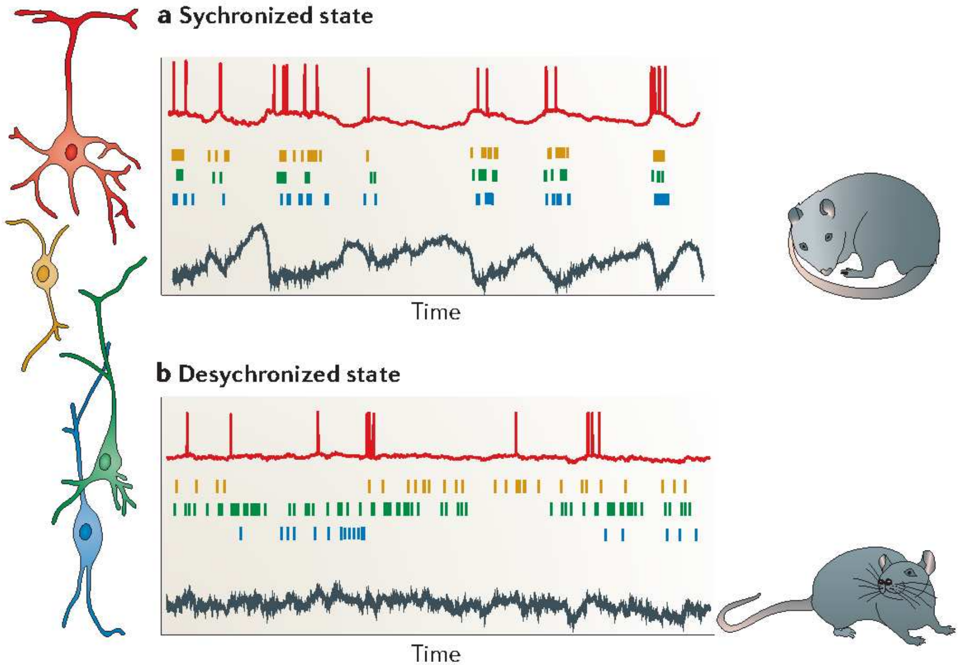
Sisyphus was condemned by Zeus for gossiping about Zeus's affair with Egira and other minor iniquities to repeat eternally his efforts, without any hope of success

Sysiphus effect in Physics

- Atomic cooling via laser irradiation (Cohen-Tannoudji)
- Low frequency fluctuations in lasers (Sano, Tartwijk, Levine, Lestra)

In the brain such endless motion can have a positive functional relevance.

Slow-Waves in the Brain



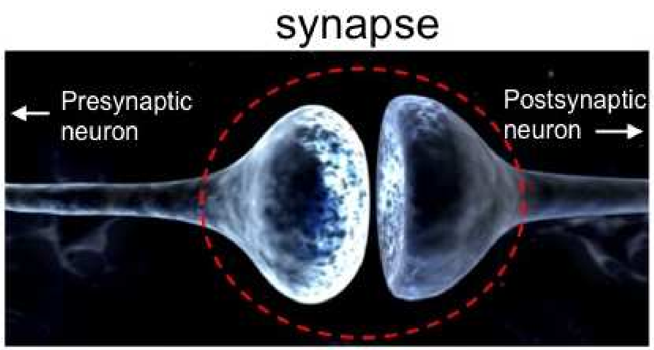
During slow-wave and REM sleep low-frequency fluctuations (LFFs) appear in the power spectrum of the Local Field Potential (LFP) (e.g the mean field activity of the neurons).

Instead in the awake animal the LFP does not show LFFs.

Fluctuating spontaneous activity has been observed in several areas of the brain: neocortex, hyppocampus,etc.

Irregular oscillations between more and less synchronized states have been revealed in the hippocampus during slow-wave sleep and this activity has been related to memory consolidation in the neocortex.

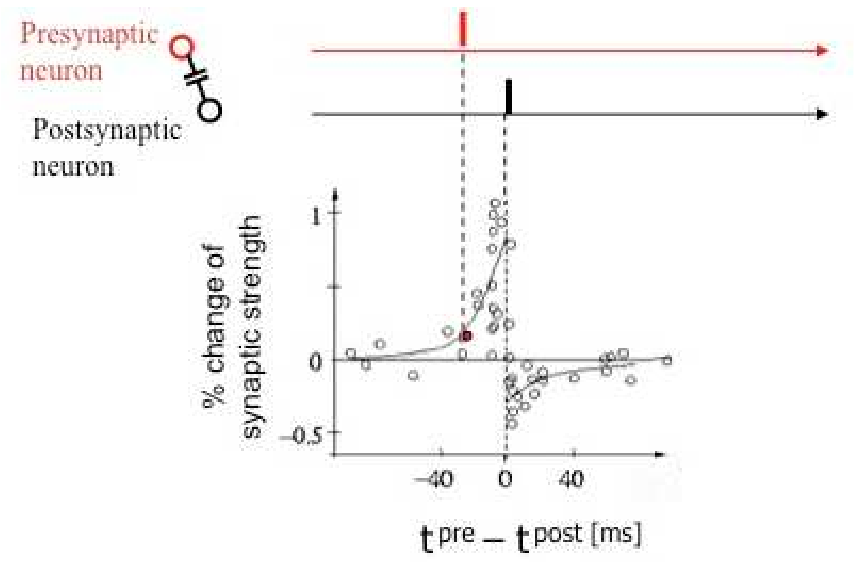
Synaptic Plasticity



- Synaptic Plasticity represents the possibility that a connection, synapse, between two neurons can modify its strength depending on the pulse transmission, reception, emission along the synaptic pathways.

- Synaptic plasticity seems to be fundamental to observe multistability in neuronal networks
  [Maistrenko et al. PRE 75, 066207 (2007); Mongillo et al. PRL 108, 158101 (2012)]
- Experimental measurements have revelead that the strengh of the synapse depends crucially on the precise spike timing of the two connected neurons - Spike-timing dependent plasticity (STDP)
- STDP and propagations delays can promote, in randomly driven networks , the emergence of states at the border between randomness and synchrony. These states resemble slow waves in the brain
  [Lubenov and Siapas, Neuron 58, 118 (2008)]

STDP



- If the presynaptic neuron fires before the postsynaptic one, the synapse is potentiated
- If the postsynaptic neuron fires before the presynaptic one, the synapse is depressed

$$\delta_{ij} = t^{pre} - t^{post}$$

w_{ij} refers to the synapse connecting i (pre) to j (post)

$$w_{ij}(t^+) = w_{ij}(t^-) + \Gamma_{ij}(t) \qquad \Gamma_{ij}(t) = \begin{cases} p[w_M - w_{ij}(t^-)]\mathrm{e}^{+\frac{\delta_{ij}}{\tau_+}} & \text{if} \quad \delta_{ij} < 0 \\ -d\, w_{ij}(t^-)\mathrm{e}^{-\frac{\delta_{ij}}{\tau_-}} & \text{if} \quad \delta_{ij} > 0 \end{cases}$$

where $d = p = 0.01$, $\tau_- = 3\tau_+ = 0.3$ and $w_M = 2$

A simple neural model

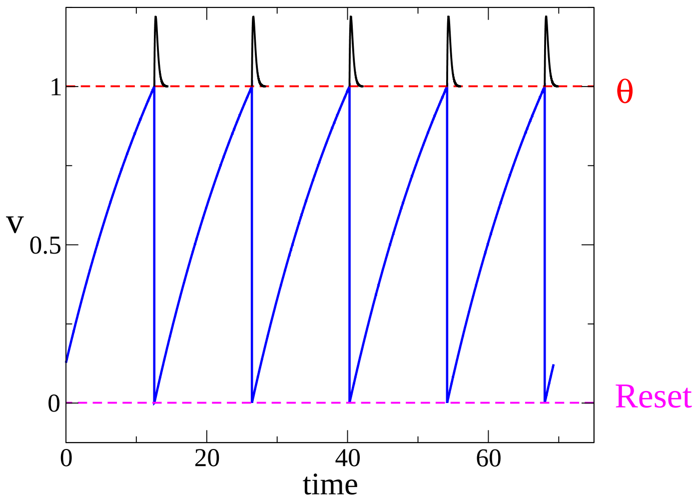
A system of N excitatory ($g > 0$) pulse-coupled leaky integrate-and-fire (LIF) neurons:

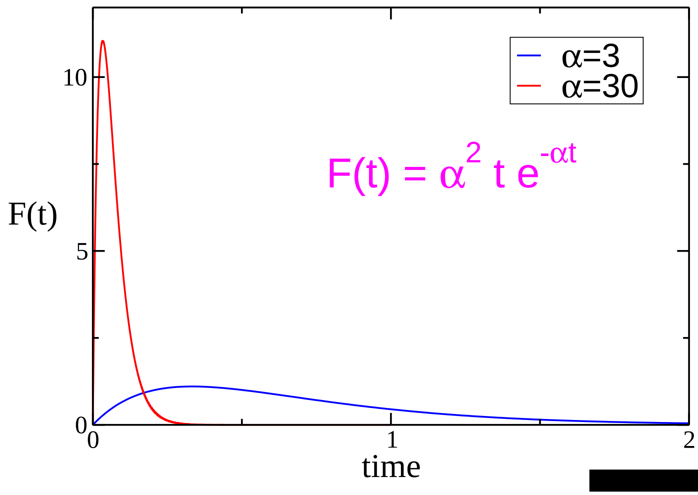
$$\dot{v}_j = I - v_j + gE(t) \quad j = 1, \dots, N$$

Supratheshold neurons with DC current $I > 1$

- If the membrane potential of a neuron v_j passes the threshold $\Theta = 0$
- a α pulse of shape $P(t) = \alpha^2 t \exp(-\alpha t)$ is sent to all the otehr neurons
- the neuron j is reset $v_j \equiv 0$

The field $E(t)$ is due to the (linear) super-position of all the past pulses

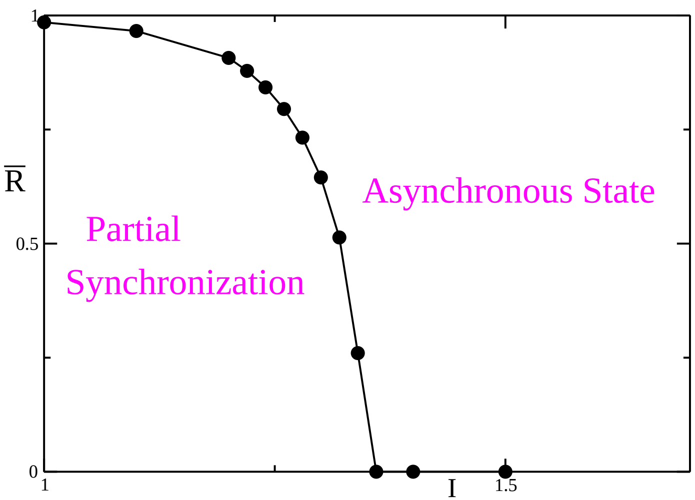


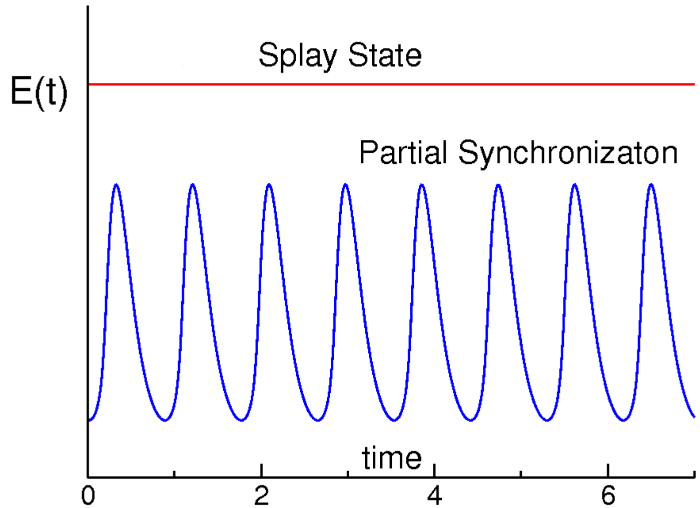


Nonplastic network

In absence of plasticity all the synaptic weights associated to the connection from the presynaptic jth neuron to the postsynaptic ith one are

$$w_{ij} = 1 \quad i \neq j$$



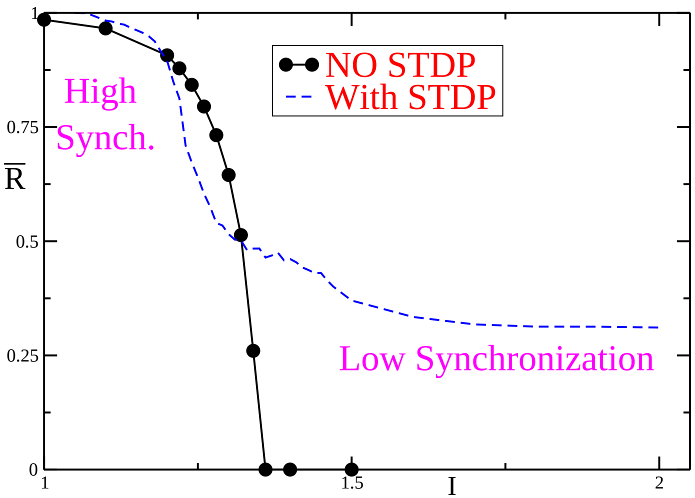


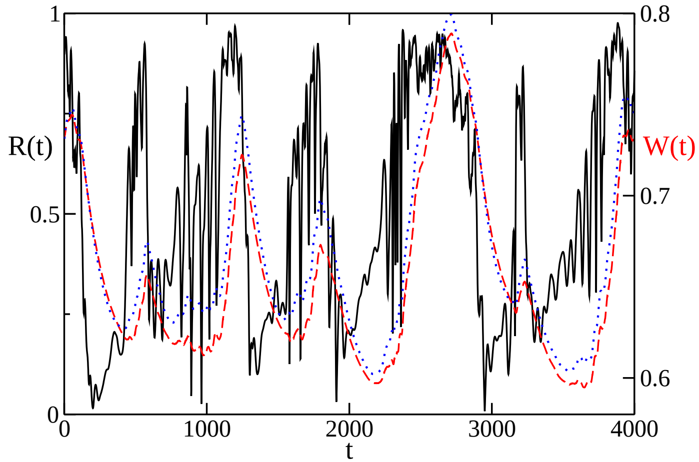
The possible dynamical regimes are

- Asynchronous regime: no collective dynamics
- Partial synchronization: regular collective oscillations

Abbott - van Vreeswiijk, Physical Review E (1993)

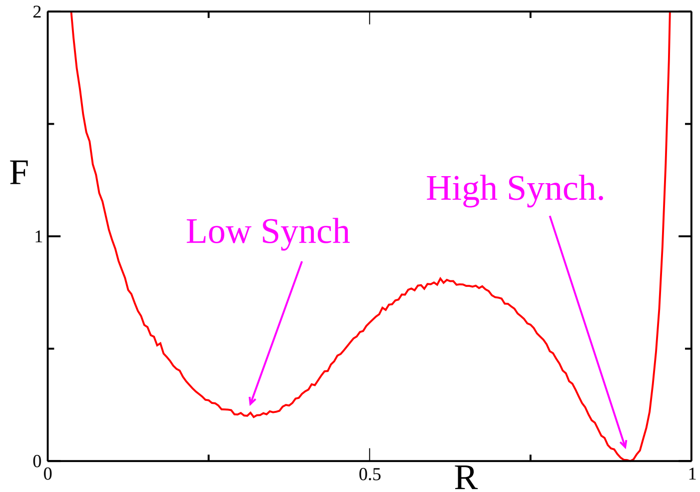
Plastic network

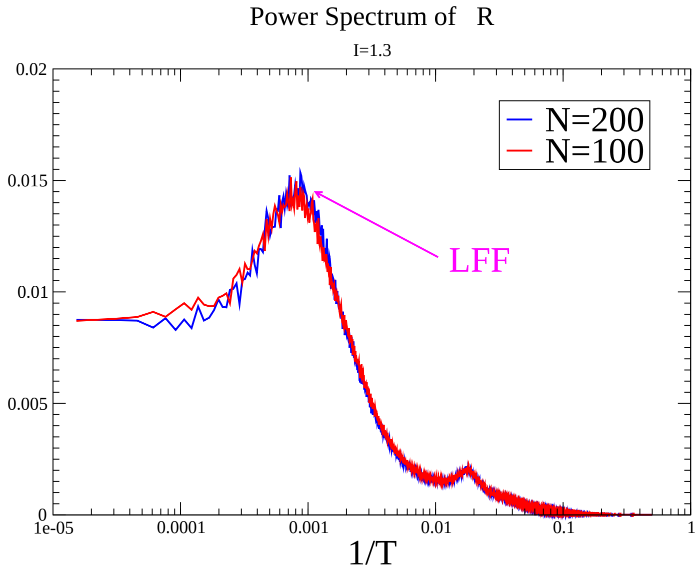




- R is a measure of the synchronization of the neurons
  - $R = 1$ fully synchronized - $R = 0$ desynchronized
  - $R < 1$ partial synchronization
- Small I Highly synchronized - Large I Low synchronization
  - For intermediate currents ($I = 1.3$) R exhibits large fluctations among $R = 0$ and $R = 1$, also W oscillates (with some delay with respect to R)
- $W(t) = \frac{1}{N(N-1)} \sum_{j,i=1}^{N} w_{ji}(t)$ Average Synaptic Weigth

Low Frequency Fluctuations



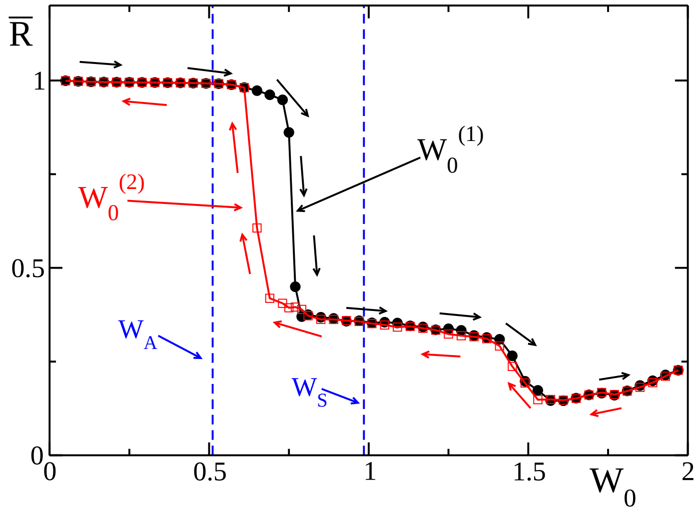


- $F(R) = -\ln P(R)$ where $P(R)$ is the PDF of R
  - F exhibits 2 minima at $R_L \simeq 0.32$ and $R_H \simeq 0.91$
  - The two minima are separated by a saddle at $R_S \simeq 0.61$
- The power spectrum of $R(t)$ reveals a peak corresponding to Low Frequency Fluctuations with average period $T \simeq 1200 - 1400$

Which is the origin of the oscillations in R ?

Constrained Simulations

How would the system evolve if the weights are decoupled from the neural dynamics ?



- We perform simulations by constraining the average weight value to W_0 by rescaling the weights during the simulation
- We first increase W_0 at steps ΔW_0 from 0 to W_M and then we decrease W_0 (duration of each simulation $T_S = 1000$)
- We observe an hysteretic transition among HS and LS
- Coexistence of HS and LS phase within the interval $[W_0^{(2)}, W_0^{(1)}]$

Mean synaptic evolution

How would the weights evolve if decoupled from the neuronal dynamics ?

Given the distribution $P(\delta)$ of the time differences δ between presynaptic and postsynaptic firing times, then we can estimate the mean field evolution of W as

$$W(t+\Delta t) = W(t) + \Gamma(t)$$

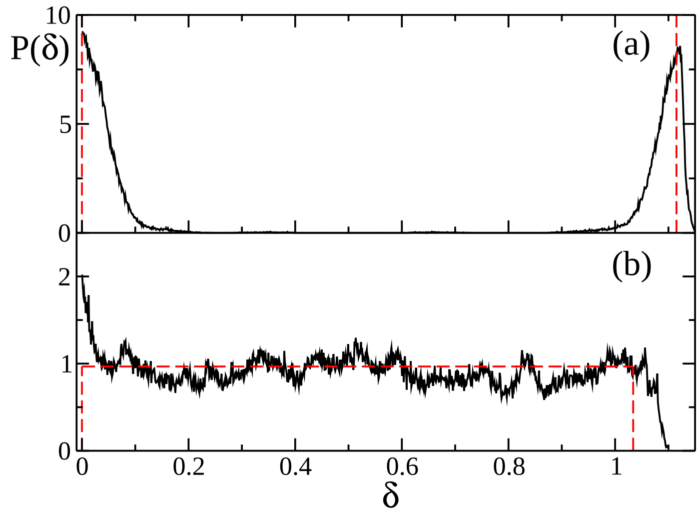
where

$$\Gamma(t) = p(w_M - W)\int_0^\infty d\delta P(\delta) \mathrm{e}^{\frac{-\delta}{\tau+}} - dW \int_{-\infty}^0 d\delta P(-\delta)\mathrm{e}^{\frac{\delta}{\tau-}}$$

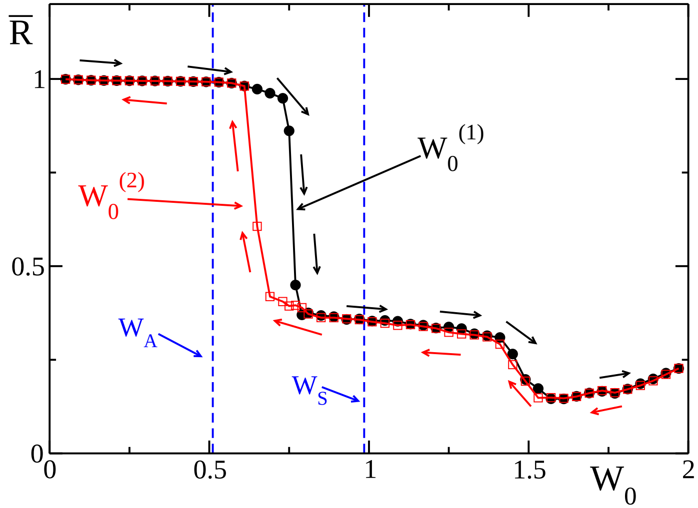
The integral can be estimated in the two limiting case of

- Asynchronous dynamics
  $P_A(\delta) = 1/T_0$
- Fully synchonized
  $P_S(\delta) = \mathcal{D}(\delta) + \mathcal{D}(\delta - T_0)$

In both cases the dynamics of W is attracted by a stable fixed point W_S (W_A)



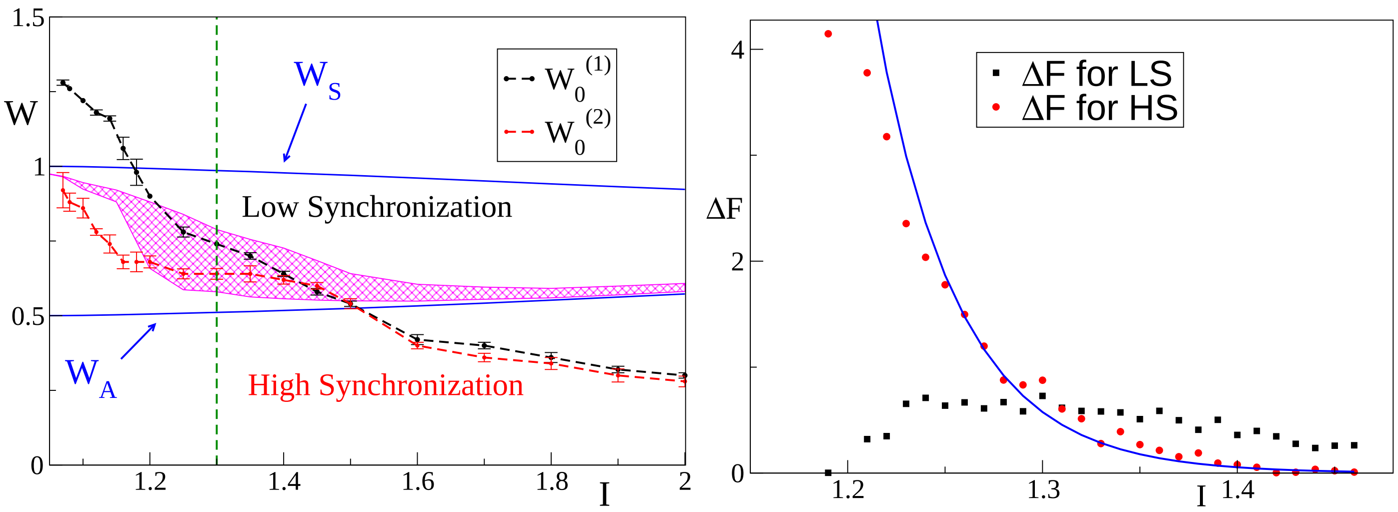
Sisyphus Effect



- In the synchronized state the weights tend toward $W_S \simeq 0.92 > W_0^{(1)}$ unfortunately this average weight corresponds to low synchronization
- In the asynchronous regime W tends towards $W_A \simeq 0.51 < W_0^{(2)}$ which is associated to a High Synchronization
- Therefore if $R \simeq 1$ implies that $W \to W_S$, whenever $W > W_0^{(1)}$ the system desynchronizes and the new attractive point is W_A, W decreases until it is smaller than $W_0^{(2)}$ and the system resynchronizes.

Thus the system oscillates endlessly

General Picture

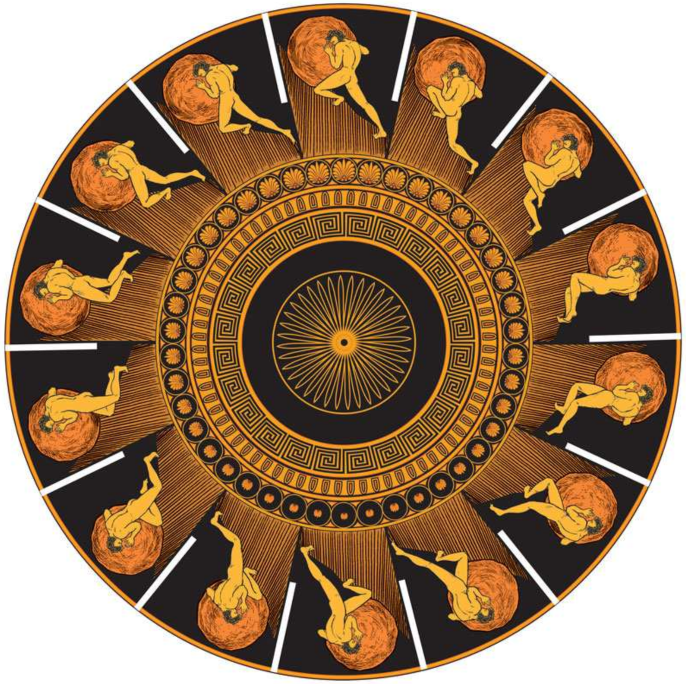


- The Sisyphus Effect should be active whenever the transition values $W_0^{(1)}$ and $W_0^{(2)}$ are both contained within the interval $[W_A, W_S]$
- From the phase diagram we expect that $F(R)$ exhibits two coexisting minima, due to the Sisyphus Effect, for $1.18 \leq I \leq 1.50$
- Indeed, this is verified since the energy barriers $\Delta F(R)$ exist only in such interval

Mikkelsen, Imparato, AT, Phys Rev Lett 110, 208101 (2013)

Conclusions

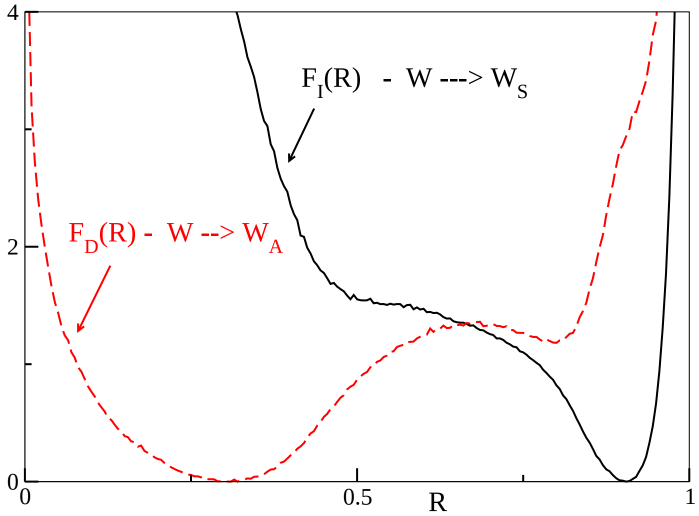
- The Sisyphus Effect should be observable in pulse coupled neural networks whenever the excitation has a desynchronizing effect
- This is in general verified for any kind of neuronal response (type I or type II) for sufficiently slow synaptic interactions
  [van Vreeswijk, Abbott, Ermentrout, J. Comp. Neuroscience (1994) -- Hansel, Mato, Meunier, Neural Comput. (1995)]
- From experimental evidences $p \simeq 2d$: Sisyphus still continues to roll the rock



(Mary Huang - Portfolio)

Sisyphus Effect in a Nutshell

- In the synchronized state the weights decreases towards the fixed point $W_S \simeq 0.92 > W_0^{(1)}$
- In the asynchronous regime W increases towards the stable fixed point $W_A \simeq 0.51 < W_0^{(2)}$



Small (large) synaptic weights tilt the landscape towards the strongly (weakly) synchronized state, in turn the induced neuronal activity increases (reduces) the weights until a tilt in the opposite direction occurs.

Thus the landscape oscillates endlessly.